

Electronic properties of quantum bars

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(February 1, 2008)

Spectrum of boson fields and two-point correlators are analyzed in a quantum bar system (a superlattice formed by two crossed interacting arrays of quantum wires), with short range interwire interaction. The standard bosonization procedure is shown to be valid, within the two wave approximation. The system behaves as a sliding Luttinger liquid in the vicinity of the Γ point, but its spectral and correlation characteristics have either 1D or 2D nature depending on the direction of the wave vector in the rest of the Brillouin zone. Due to interwire interaction, unperturbed states, propagating along the two arrays of wires, are always mixed, and the transverse components of the correlation functions do not vanish. This mixing is especially strong around the diagonals of the Brillouin zone, where transverse correlators have the same order of magnitude as the longitudinal ones.

I. INTRODUCTION AND OVERVIEW

Diverse $D - 1$ dimensional objects embedded in D dimensional structures were recently investigated experimentally and analyzed theoretically. Rubbers and various percolation networks are examples of such disordered $D - 1$ objects, whereas self-organized stripes in oxocuprates, manganites, nanotube ropes and Quantum Hall systems are examples of periodic structures of this kind. In some cases, the effective dimensions of such ordered structures may be intermediate *e.g.* between $D = 1$ and $D = 2$. They are especially promising candidates for studying novel electronic correlation properties, which, in particular, are relevant for the search of Luttinger liquid (LL) finger-prints in two dimensions. This challenging idea is motivated by noticing some unusual properties of electrons in Cu-O planes in High- T_c materials¹. However, the Fermi liquid state seems to be rather robust in two dimensions. In this respect, a 2D system of weakly coupled 1D quantum wires²⁻⁴ looks promising. Indeed, a theoretical analysis of stable LL phases was recently presented for a system consisting of coupled parallel quantum wires⁵⁻⁷ and for 3D stacks of sheets of such wires in parallel and crossed orientations⁸. In most of these cases, the interaction between the parallel quantum wires is assumed to be perfect along the wire⁸, whereas the interaction between the modes generated in different wires depends only on the inter-wire distance. Along these lines, generalization of the LL theory for quasi 2D (and even 3D) systems is reported in Ref. 8 where the interaction between two crossed arrays of parallel quantum wires forming some kind of a network, depends on the distance between points belonging to different arrays. As a result, the grid of crossed arrays retains its LL properties for the propagation along both subsets of parallel wires, whereas cross-correlations remained non-singular. This LL structure can be interpreted as a quantum analog of a classical sliding phases of coupled XY chains⁹. A special case of 2D grid where the crossed wires are coupled by tunneling interaction is considered in Refs. 4,10

In the present paper, a different course is elaborated. We ask the question whether it is possible to encode *both 1D and 2D electron liquid regimes in the same system within the same energy scale*. In order to unravel the pertinent physics we consider a grid with *short-range inter-wire interaction*. This approximation might look shaky if applied for crossed stripe arrays in the cuprates. On the other hand, it seems natural for 2D grids of nanotubes^{11,12}, or artificially fabricated bars of quantum wires with grid periods $a_{1,2}$ which exceed the lattice spacing of a single wire or the diameter of a nanotube. It will be demonstrated below that the short-range interaction with radius $r_0 \ll a_{1,2}$ turns out to be effectively weak. Therefore, such a quantum bar (QB) retains the 1D LL character for the motion along the wires similarly to the case considered in Ref. 8. At the same time, however, the boson mode propagation along the diagonals of the grid is also feasible. This process is essentially a two-dimensional one, as well as the shape of the Brillouin zone and equipotential surfaces in the reciprocal QB lattice.

Before developing the formalism, a few words about the main assumptions are in order. Our attention here is mainly focused on charge modes, so it is assumed that there is a gap for spin excitations. Next, we are mainly interested in electronic properties of QB which are not related to simple charge instabilities like commensurate CDW, so that the (for simplicity equal) periods $a_1 = a_2 = a$ are supposed to be incommensurate with the lattice spacing. The Brillouin zone (BZ) of the QB superlattice is two-dimensional, and the nature of excitations propagating in this BZ is determined by Bragg interference of modes with the superlattice wave vector. This interference (Umklapp processes) is, of course destructive for LL excitations with both wave vector components close to multiple integers of $2\pi/a$. However, in case of weak scattering $Vag/\hbar v_F \ll 1$, where V is the interwire interaction strength, g is the usual LL parameter and v_F is the Fermi velocity, only two-wave interference processes near the boundaries of the BZ are significant.

One can then hope that the harmonic boson modes survive in the major part of the BZ, and that the Hamiltonian of the QB might still be diagonalized without losing the main characteristic features of the LL physics.

II. FORMALISM: HAMILTONIAN AND MAIN APPROXIMATIONS

We consider a 2D periodic grid consisting of two arrays of 1D quantum wires of length L oriented along unit vectors $\mathbf{e}_{1,2}$ with an angle φ between them. The full Hamiltonian of the system is,

$$H = H_1 + H_2 + H_{int}. \quad (1)$$

The Hamiltonian H_i describes the 1D boson field in the i -th array of wires ($i = 1, 2$)

$$H_1 = \frac{\hbar v}{2} \sum_{n_2} \int_{-L/2}^{L/2} dx_1 \left\{ g\pi_1^2(x_1, n_2a) + \frac{1}{g} (\partial_{x_1} \theta_1(x_1, n_2a))^2 \right\},$$

$$H_2 = \frac{\hbar v}{2} \sum_{n_1} \int_{-L/2}^{L/2} dx_2 \left\{ g\pi_2^2(n_1a, x_2) + \frac{1}{g} (\partial_{x_2} \theta_2(n_1a, x_2))^2 \right\}.$$

Here n_i enumerates the wires in the i -th array, x_i is a 1D continuous coordinate along \mathbf{e}_i , while (θ_i, π_i) are the conventional canonically conjugate boson fields (see, e.g., Ref.[13]). The Fermi velocities $v_{F1,2} = v$ and the LL parameters $g_{1,2} = g$ are taken to be the same for both arrays. The modulus of each component of a quasimomentum in the first BZ does not exceed $Q/2$ where $Q = 2\pi/a$. Generalization to the case of different parameters v_i, g_i, a_i is straightforward.

The interwire interaction results from a short-range contact capacitive coupling in the crosses of the bar,

$$H_{int} = \sum_{n_1, n_2} \int_{-L/2}^{L/2} dx_1 dx_2 V(x_1 - n_1a, n_2a - x_2) \times \rho_1(x_1, n_2a) \rho_2(n_1a, x_2). \quad (2)$$

Here $\rho_i(\mathbf{r})$ are density operators, and $V(\mathbf{r}_1 - \mathbf{r}_2)$ is a short-range interwire interaction. Physically, it represents a highly screened Coulomb electron-electron interaction between two charges located at the points $\mathbf{r}_1 = (x_1, n_2a)$ and $\mathbf{r}_2 = (n_1a, x_2)$. In what follows we use an interaction of the form,

$$V(\mathbf{r}) = \frac{V_0}{2} \Phi\left(\frac{x_1}{r_0}, \frac{x_2}{r_0}\right).$$

Here V_0 is the coupling strength and r_0 is the screening radius. The function $\Phi(\xi_1, \xi_2)$ vanishes for $|\xi_{1,2}| \geq 1$, and is normalized by condition $\Phi(0, 0) = 1$. For simplicity it is taken to be an even function of its two variables. In terms of boson field operators θ_i , the interaction acquires the following form,

$$H_{int} = V_0 \sum_{n_1, n_2} \int_{-L/2}^{L/2} dx_1 dx_2 \Phi\left(\frac{x_1 - n_1a}{r_0}, \frac{n_2a - x_2}{r_0}\right) \times \partial_{x_1} \theta_1(x_1, n_2a) \partial_{x_2} \theta_2(n_2a, x_2). \quad (3)$$

Anticipating a Fourier expansion we introduce a 2D basis of periodic Bloch functions which is constructed in terms of 1D Bloch functions for non-interacting QB,

$$\Psi_{p,p',\mathbf{q}}(\mathbf{r}) = \psi_{p,q_1}(x_1) \psi_{p',q_2}(x_2), \quad (4)$$

where

$$\psi_{p,q}(x) = \frac{1}{\sqrt{L}} e^{iqx} f_p(q, x),$$

and

$$f_p(q, x) = \exp\left\{i \operatorname{sign}(q) (-1)^{p+1} \left[\frac{p}{2}\right] Qx\right\}.$$

Here $p, p' = 1, 2, \dots$, are the band numbers in a reduced BZ, and $\mathbf{q} = \{q_1, q_2\}$ is the crystal quasimomentum, ($|q_i| \leq Q/2$). In momentum representation the full Hamiltonian (1) then acquires the form,

$$H = \frac{\hbar v g}{2a} \sum_{i=1}^2 \sum_p \sum_{\mathbf{q}} \pi_{ip\mathbf{q}}^+ \pi_{ip\mathbf{q}} + \frac{\hbar}{2vga} \sum_{ii'=1}^2 \sum_{pp'} \sum_{\mathbf{q}} W_{ipip'p'\mathbf{q}} \theta_{ip\mathbf{q}}^+ \theta_{i'p'\mathbf{q}}, \quad (5)$$

with matrix elements for interwire coupling given by,

$$W_{ipip'p'\mathbf{q}} = \omega_{ip\mathbf{q}} \omega_{i'p'\mathbf{q}} [\delta_{ii'} \delta_{pp'} + \alpha_{ipip'p'\mathbf{q}} (1 - \delta_{ii'})].$$

Here

$$\omega_{ip\mathbf{q}} = v \left(\left[\frac{p}{2}\right] Q + (-1)^{p+1} |q_i| \right), \quad (6)$$

are eigenfrequencies of the “unperturbed” 1D mode pertaining to an array i , band p and quasimomentum \mathbf{q} . The coefficients

$$\alpha_{ipip'p'\mathbf{q}} = \operatorname{sign}(q_1 q_2) (-1)^{p+p'} \frac{g V_0 r_0^2}{\hbar v a} \Phi_{ipip'p'\mathbf{q}}, \quad (7)$$

are proportional to the dimensionless Fourier component of the interaction strengths

$$\begin{aligned} \Phi_{1p2p'\mathbf{q}} &= \int d\xi_1 d\xi_2 \Phi(\xi_1, \xi_2) e^{-ir_0(q_1\xi_1 + q_2\xi_2)} \times \\ &\times f_p^*(q_1, r_0\xi_1) f_{p'}^*(q_2, r_0\xi_2) = \Phi_{2p'1p\mathbf{q}}. \end{aligned} \quad (8)$$

The Hamiltonian (5) describes a system of coupled harmonic oscillators, which can be *exactly* diagonalized with the help of a certain canonical linear transformation (note that due to spatial periodicity, it is already diagonal with respect to the quasimomentum \mathbf{q}). The diagonalization procedure is, nevertheless, rather cumbersome due to the mixing of states belonging to different bands and arrays. However, it is seen from Eq.(7) that in the case $r_0 \ll a$ the dimensionless interaction α becomes effectively weak (numerical estimates are given below) and a perturbation approach is applicable. In this limit, the systematics of unperturbed levels and states is grossly conserved, at least in the low energy region corresponding to the first few bands. Indeed, as it follows from the unperturbed dispersion law (6), the inter-band mixing is significant only along the high symmetry directions in the first BZ. The effect of interband interactions can be accounted for perturbatively in the rest of the BZ. Moreover, the mixing between modes within the same energy band is strong for waves with quasimomenta close to the diagonal of the first BZ. Away from the diagonal, this mixing effect can also be calculated perturbatively.

In second order of perturbation theory the above mentioned canonical transformation results in the following renormalized field operators and the corresponding renormalized eigenfrequencies for the first array:

$$\begin{aligned} \tilde{\theta}_{1p\mathbf{q}} &= \left(1 - \frac{1}{2}\beta_{1p\mathbf{q}}\right) \theta_{1p\mathbf{q}} + \\ &+ \sum_{p'} \frac{\alpha_{1p2p'\mathbf{q}} \omega_{1p\mathbf{q}} \omega_{2p'\mathbf{q}}}{\omega_{1p\mathbf{q}}^2 - \omega_{2p'\mathbf{q}}^2} \theta_{2p'\mathbf{q}}, \end{aligned} \quad (9)$$

where

$$\beta_{1p\mathbf{q}} = \sum_{p'} \left(\frac{\alpha_{1p2p'\mathbf{q}} \omega_{1p\mathbf{q}} \omega_{2p'\mathbf{q}}}{\omega_{1p\mathbf{q}}^2 - \omega_{2p'\mathbf{q}}^2} \right)^2, \quad (10)$$

and

$$\tilde{\omega}_{1p\mathbf{q}}^2 = \omega_{1p\mathbf{q}}^2 \left[1 + \sum_{p'} \frac{\alpha_{1p2p'\mathbf{q}}^2 \omega_{2p'\mathbf{q}}^2}{\omega_{1p\mathbf{q}}^2 - \omega_{2p'\mathbf{q}}^2} \right].$$

Corresponding formulas for the second array are obtained by replacing $1p \rightarrow 2p$, and $1p' \rightarrow 2p'$.

For quasimomenta lying off the diagonal of the first BZ, all terms in these equations are non-singular. Therefore, the applicability of perturbation theory is related to the convergence of the series on the right hand side of Eq.(10). Away from the first BZ boundary ($|\mathbf{q}| \ll Q/2$) the following estimation is valid,

$$\begin{aligned} \frac{\omega_{ip\mathbf{q}}^2}{\omega_{ip\mathbf{q}}^2 - \omega_{i'1\mathbf{q}}^2} &\approx 1 + O\left(\frac{q_{i'}^2}{([p/2]Q)^2}\right), \\ (i, i') &= (1, 2), (2, 1); \quad p > 1. \end{aligned}$$

Therefore, the correction can be approximately estimated as $\omega_{1p\mathbf{q}}^2 S_{\mathbf{q}}$ with

$$S_{\mathbf{q}} = \sum_p \alpha_{112p\mathbf{q}}^2 = \left(\frac{V_0 g r_0^2}{\hbar v a}\right)^2 \sum_p \Phi_{112p\mathbf{q}}^2. \quad (11)$$

For a short-range interaction, $r_0 \ll a$, $\Phi_{ipi'p'\mathbf{q}}$ is a smooth function of p and p' . Therefore, the sum over p in Eq.(11) can be replaced by an integral over the extended BZ with wave vector \mathbf{k} whose components are

$$k_i = q_i + \text{sign}(q_i) (-1)^{p_i+1} [p_i/2] Q.$$

For $|\mathbf{q}| \ll Q$ one gets $S_{\mathbf{q}} = S_0(1 + o(1))$ where,

$$S_0 = \left(\frac{V_0 g r_0^2}{\hbar v a}\right)^2 \frac{a}{2\pi} \int dk \tilde{\Phi}_{0k}^2,$$

and

$$\tilde{\Phi}_{k_1 k_2} = \int d\xi_1 d\xi_2 \Phi(\xi_1, \xi_2) e^{-ir_0(k_1\xi_1 + k_2\xi_2)}.$$

Finally, one arrives at the estimate,

$$S_0 \approx \left(\frac{V_0 g \Phi_0}{\hbar v}\right)^2 \frac{r_0^3}{a}, \quad (12)$$

where

$$\Phi_0^2 = \int d\xi d\xi_1 d\xi_2 \Phi(\xi_1, \xi) \Phi(\xi_2, \xi).$$

In the case of QB formed by nanotubes, V_0 is the Coulomb interaction screened at a distance of the order of the nanotube radius¹⁴ R_0 , so that $V_0 \approx e^2/R_0$ and $r_0 \approx R_0$. Therefore,

$$S_0 = \left(\frac{e^2 g \Phi_0}{\hbar v}\right)^2 \frac{R_0}{a}. \quad (13)$$

For carbon nanotubes one has¹¹ $v \approx 8 \cdot 10^7$ cm/sec and $g \approx 1/3$. Then, Φ_0 can be calculated e.g. for a Gaussian model,

$$\Phi(\xi_1, \xi_2) \propto e^{-(\xi_1^2 + \xi_2^2 - 2\xi_1\xi_2 \cos \varphi)}.$$

Taking for example $\varphi = \pi/3$, one finds that interaction is effectively weak if $a \gg 4R_0$. Therefore even slightly rarefied QB already satisfies the desired condition. Thus, the dimensionless coupling constant α (7) is a small parameter of the theory. Further calculations are carried out within an accuracy $o(\alpha^2)$. Yet, in the equations presented below, only the first non-vanishing representative terms are retained (just for the sake of brevity).

Consider now modes with quasi-momenta near the diagonal of the first Brillouin zone, but away from its boundary. In this case, the frequencies of the modes belonging to the same band coincide, $\omega_{1p\mathbf{q}} = \omega_{2p\mathbf{q}} \equiv \omega_{p\mathbf{q}}$. Therefore the modes are strongly mixed:

$$\begin{aligned}\tilde{\theta}_{1p\mathbf{q}} &\approx \frac{1}{\sqrt{2}} (\theta_{1p\mathbf{q}} + \theta_{2p\mathbf{q}}), \\ \tilde{\theta}_{2p\mathbf{q}} &\approx \frac{1}{\sqrt{2}} (-\theta_{1p\mathbf{q}} + \theta_{2p\mathbf{q}}).\end{aligned}\quad (14)$$

The corresponding eigenfrequencies are shifted from their bare values already in the first order in α

$$\begin{aligned}\tilde{\omega}_{1p\mathbf{q}}^2 &\approx \omega_{p\mathbf{q}}^2 (1 + \alpha_{1p2p\mathbf{q}}), \\ \tilde{\omega}_{2p\mathbf{q}}^2 &\approx \omega_{p\mathbf{q}}^2 (1 - \alpha_{1p2p\mathbf{q}}).\end{aligned}\quad (15)$$

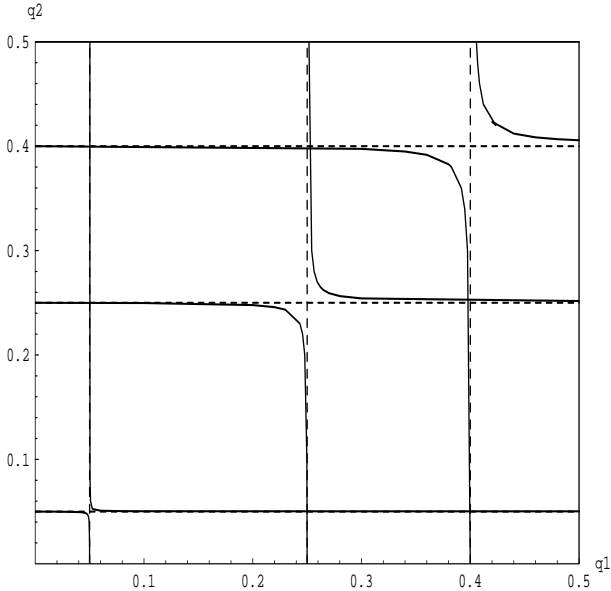


FIG. 1. Solid and dashed lines of equal energy for interacting and noninteracting systems respectively.

These results show that the quantum states of the 2D quantum bar conserve the quasi 1D character of the Luttinger-like liquid in the major part of momentum space, and that the 2D effects can be calculated within the framework of perturbation theory. However, it follows from Eqs.(14) that the bosons with quasimomenta close to the diagonal of the first BZ are the strongly mixed bare 1D bosons. These excitations are essentially two-dimensional, and therefore the lines of equal energy in this part of BZ are modified by the 2D interaction (1). It is clearly seen that deviations from linearity occur only in a small part of the BZ. However, due to the absence of charge transfer, the Fermi surface of QB is still a “cross” obtained by superposition of two mutually perpendicular stripes of width $|2k_F|$ around the lines $k_x = 0, k_y = 0$ (cf. Ref. 4). The crossover from LL to FL behavior around isolated points of the BZ due to a single-particle hybridization (tunneling) for Fermi excitations was noticed

in Refs. 4,10, where a mesh of horizontal and vertical stripes in superconducting cuprates was studied.

III. CORRELATIONS AND OBSERVABLES

The structure of the energy spectrum analyzed above predetermines optical and transport properties of the QB. Let us consider an optical conductivity $\sigma_{ii'}(\mathbf{q}, \omega)$ whose spectral properties are given by a current-current correlator

$$\sigma_{ii'}(\mathbf{q}, \omega) = \text{Re} \left(\frac{1}{\omega} \int_0^\infty dt e^{i\omega t} \left\langle \left[j_{i1\mathbf{q}}(t), j_{i'1\mathbf{q}}^\dagger(0) \right] \right\rangle \right). \quad (16)$$

Here $j_{ip\mathbf{q}} = \sqrt{2}vg\pi_{ip\mathbf{q}}$ is a current operator. For simplicity we restrict ourselves to the first band. For non-interacting wires, the current-current correlator is reduced to the conventional LL expression¹⁵,

$$\left\langle \left[j_{i1\mathbf{q}}(t), j_{i'1\mathbf{q}}^\dagger(0) \right] \right\rangle_0 = -2ivg\omega_{i1\mathbf{q}} \sin(\omega_{i1\mathbf{q}}t) \delta_{ii'} \quad (17)$$

with a metallic Drude peak

$$\sigma_{ii'}(\mathbf{q}, \omega > 0) = \pi vg \delta(\omega - \omega_{i1\mathbf{q}}) \delta_{ii'}.$$

For interacting wires, where $\alpha_{ipi'p'\mathbf{q}} \neq 0$, the correlators may be easily calculated after diagonalization of the Hamiltonian (5) by the transformations (9) for \mathbf{q} off the diagonal of the first BZ, or by the transformation (14) for \mathbf{q} lying on the diagonal of the BZ.

Consider first the optical conductivity for \mathbf{q} far from the diagonal of the first BZ. In this case, the transformations for the field momenta can be obtained in a similar manner to the transformations (9) for the field coordinates. As a result, one has:

$$\begin{aligned}\left\langle \left[j_{i1\mathbf{q}}(t), j_{i'1\mathbf{q}}^\dagger(0) \right] \right\rangle &\approx \\ &- 2ivg (1 - \beta_{\mathbf{q}}^2) \tilde{\omega}_{i1\mathbf{q}} \sin(\tilde{\omega}_{i1\mathbf{q}}t) - \\ &- 2ivg\beta_{\mathbf{q}}^2 \tilde{\omega}_{2p\mathbf{q}} \sin(\tilde{\omega}_{2p\mathbf{q}}t),\end{aligned}\quad (18)$$

$$\begin{aligned}\left\langle \left[j_{i1\mathbf{q}}(t), j_{i'2\mathbf{q}}^\dagger(0) \right] \right\rangle &\approx -2ivg\beta_{\mathbf{q}} \times \\ &\times (\tilde{\omega}_{i1\mathbf{q}} \sin(\tilde{\omega}_{i1\mathbf{q}}t) - \tilde{\omega}_{2p\mathbf{q}} \sin(\tilde{\omega}_{2p\mathbf{q}}t)),\end{aligned}$$

where $\alpha_{\mathbf{q}} \equiv \alpha_{1121\mathbf{q}}$ and

$$\beta_{\mathbf{q}} = \frac{\alpha_{\mathbf{q}}\omega_{11\mathbf{q}}\omega_{21\mathbf{q}}}{\omega_{11\mathbf{q}}^2 - \omega_{21\mathbf{q}}^2}.$$

Then one obtains

$$\begin{aligned}\sigma_{11}(\mathbf{q}, \omega > 0) &\approx \pi vg (1 - \beta_{\mathbf{q}}^2) \delta(\omega - \tilde{\omega}_{11\mathbf{q}}) + \\ &+ \pi vg \beta_{\mathbf{q}}^2 \delta(\omega - \tilde{\omega}_{2p\mathbf{q}}),\end{aligned}\quad (19)$$

$$\sigma_{12}(\mathbf{q}, \omega > 0) \approx \pi v g \beta_{\mathbf{q}} \times [\delta(\omega - \tilde{\omega}_{11\mathbf{q}}) - \delta(\omega - \tilde{\omega}_{21\mathbf{q}})]. \quad (20)$$

The longitudinal optical conductivity (19) (i.e. the conductivity within a given set of wires) has its main peak at the frequency $\tilde{\omega}_{11\mathbf{q}} \approx v|q_1|$, corresponding to the first band of the pertinent array, and an additional weak peak at the frequency $\tilde{\omega}_{21\mathbf{q}} \approx v|q_2|$, corresponding to the first band of a complementary array. It contains also a set of weak peaks at frequencies $\tilde{\omega}_{2p\mathbf{q}} \approx [p/2]vQ$ ($p = 2, 3, \dots$), omitted in Eq.(19) and corresponding to the contribution from higher bands of the complementary array. At the same time, a second observable becomes relevant, namely, the transverse optical conductivity (20). It is proportional to the interaction strength and has two peaks at frequencies $\tilde{\omega}_{11\mathbf{q}}$ and $\tilde{\omega}_{21\mathbf{q}}$ in the first bands of both sets of wires. For $|\mathbf{q}| \rightarrow 0$, Eq.(18) reduces to that for an array of noninteracting wires, and the transverse optical conductivity vanishes.

In case where the quasimomenta \mathbf{q} belong to the diagonal of the first BZ, the transformations for the field momenta are similar in form to Eqs. (14). The current-current correlation functions have the form

$$\begin{aligned} & \langle [j_{11\mathbf{q}}(t), j_{11\mathbf{q}}^\dagger(0)] \rangle \approx \\ & -ivg [\tilde{\omega}_{11\mathbf{q}} \sin(\tilde{\omega}_{11\mathbf{q}}t) + \tilde{\omega}_{21\mathbf{q}} \sin(\tilde{\omega}_{21\mathbf{q}}t)], \end{aligned} \quad (21)$$

$$\begin{aligned} & \langle [j_{1\mathbf{q}}(t), j_{2\mathbf{q}}^\dagger(0)] \rangle \approx \\ & -ivg (\tilde{\omega}_{11\mathbf{q}} \sin(\tilde{\omega}_{11\mathbf{q}}t) - \tilde{\omega}_{21\mathbf{q}} \sin(\tilde{\omega}_{21\mathbf{q}}t)), \end{aligned}$$

and the optical conductivity is estimated as,

$$\sigma_{11}(\mathbf{q}, \omega > 0) \approx \frac{\pi v g}{2} [\delta(\omega - \tilde{\omega}_{11\mathbf{q}}) + \delta(\omega - \tilde{\omega}_{21\mathbf{q}})], \quad (22)$$

$$\sigma_{12}(\mathbf{q}, \omega > 0) \approx \frac{\pi v g}{2} [\delta(\omega - \tilde{\omega}_{11\mathbf{q}}) - \delta(\omega - \tilde{\omega}_{21\mathbf{q}})]. \quad (23)$$

Here, the index i of the array is omitted because the frequencies for both arrays coincide, $\omega_{1p\mathbf{q}} = \omega_{2p\mathbf{q}}$. The longitudinal optical conductivity (22) has a split double peak at frequencies $\tilde{\omega}_{11\mathbf{q}}$ and $\tilde{\omega}_{21\mathbf{q}}$, instead of a single peak. Again, a series of weak peaks (omitted in the r.h.s. of Eq.(22)) occurs at frequencies $\omega_{p\mathbf{q}}$ corresponding to contribution from higher bands $p = 2, 3, 4, \dots$. The transverse optical conductivity (23), similarly to the non-diagonal case (20), has a split double peak at frequencies $\tilde{\omega}_{11\mathbf{q}}$ and $\tilde{\omega}_{21\mathbf{q}}$.

One of the main effects specific for a QB is the appearance of non-zero transverse momentum-momentum correlation function. In space-time coordinates (\mathbf{x}, t) its representation reads,

$$G_{12}(\mathbf{x}, t) = \langle [\pi_1(x_1, 0; t), \pi_2(0, x_2; 0)] \rangle. \quad (24)$$

This function describes the momentum response at the point $(0, x_2)$ of the second array at the moment t caused by initial ($t = 0$) perturbation at the point $(x_1, 0)$ of the first array. Standard calculations similar to those described above, lead to the following expression,

$$G_{12}(\mathbf{x}; t) = \frac{V_0 r_0^2}{2\pi \hbar v a} \int_0^\infty dk_1 dk_2 \tilde{\Phi}_{k_1 k_2} k_1 k_2 \times \sin(k_1 x_1) \sin(k_2 x_2) \frac{k_2 \sin(k_2 vt) - k_1 \sin(k_1 vt)}{k_2^2 - k_1^2}.$$

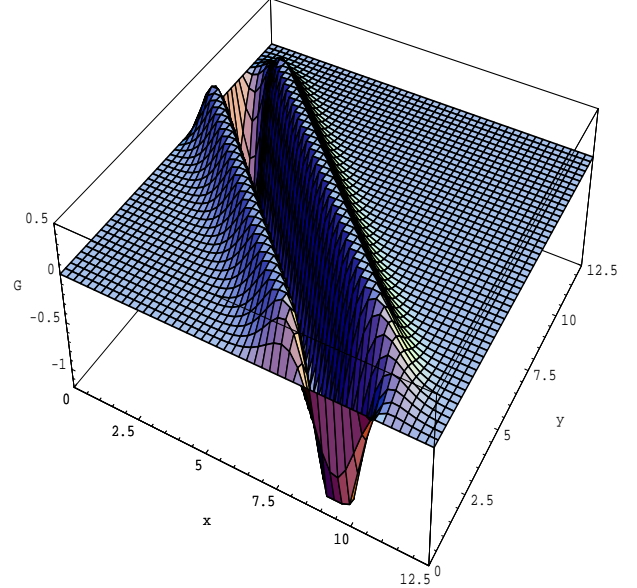


FIG. 2. The transverse correlation function $G_{12}(x_1, x_2; t)$ for $r_0 = 1$ and $vt = 10$.

This correlator is shown in Fig 2 for $x_{1,2} > 0$. Here the non-zero response corresponds to the line determined by the obvious kinematic condition $x_1 + x_2 = vt$. The finiteness of the interaction radius slightly spreads this peak and changes its profile.

Further manifestation of the 2D character of QB system is related to a possible periodic energy transfer between the two arrays of wires. Consider an initial perturbation which, in the system of *non*-interacting arrays, corresponds to a plane wave propagating within the first array along the \mathbf{e}_1 direction,

$$\begin{aligned} \langle \theta_1(x_1, n_2 a; t) \rangle &= \frac{v \rho_0}{\sqrt{2} \omega_{11\mathbf{q}}} \sin(q_1 x_1 + q_2 n_2 a - \omega_{11\mathbf{q}} t), \\ \langle \theta_2(n_1 a, x_2; t) \rangle &= 0, \end{aligned} \quad (25)$$

(ρ_0 is the charge density amplitude). If the wave vector \mathbf{q} , satisfying the condition $|\mathbf{q}| \ll Q/2$, is not close to the first BZ diagonal, weak interwire interaction α slightly changes the $\langle \theta_1 \rangle$ component and leads to the appearance of a small $\langle \theta_2 \rangle \sim \alpha$ component. But for \mathbf{q} lying on the diagonal, both components within the main approximation have the same order of magnitude

$$\theta_1(x_1, n_2a; t) = \frac{v\rho_0}{\sqrt{2}\omega_{1\mathbf{q}}} \cos\left(\frac{1}{2}\alpha_{\mathbf{q}}\omega_{1\mathbf{q}}t\right) \times \sin(q_1x_1 + q_2n_2a - \omega_{1\mathbf{q}}t), \quad (26)$$

$$\theta_2(n_1a, x_2; t) = \frac{v\rho_0}{\sqrt{2}\omega_{1\mathbf{q}}} \sin\left(\frac{1}{2}\alpha_{\mathbf{q}}\omega_{1\mathbf{q}}t\right) \times \cos(q_1n_1a + q_2x_2 - \omega_{1\mathbf{q}}t).$$

This corresponds to a 2D propagation of a plane wave with wave vector \mathbf{q} , *modulated* by a “slow” frequency $\sim \alpha\omega$. As a result, an energy is periodically transferred from one array to another during a long period $T \sim (\alpha\omega)^{-1}$. These peculiar “Rabi oscillations” may be considered as one of the fingerprints of the physics exposed in QB systems.

IV. CONCLUSION

In conclusion, we have demonstrated that the energy spectrum of QB shows the characteristic properties of LL at $|q|, \omega \rightarrow 0$, but at finite \mathbf{q} the density and momentum waves may have either 1D or 2D character depending on the direction of the wave vector. Due to inter-wire interaction, unperturbed states, propagating along the two arrays are always mixed, and transverse components of correlation functions do not vanish. For quasi-momentum lying on the diagonal of the Brillouin zone, such a mixing is strong and transverse correlators have the same order of magnitude as the longitudinal ones.

V. ACKNOWLEDGEMENT

S.G. and K.K. are indebted to L.Gorelik, M. Jonson, I. Krive, and R. Shekhter for numerous helpful discussions. They also thank Chalmers Technical University, where this work was started, for hospitality and support. This research is supported in part by grants from the Israeli Science foundations the DIP German Israeli cooperation program and the U.S Israel BSF program.

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